Binary particle swarm optimisation-based optimal substation coverage algorithm for phasor measurement unit installations in practical systems

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Abstract: Phasor measurement units (PMUs) play an important role in the wide-area monitoring and protection of modern power systems. Historically, their deployment was limited by the prohibitive cost of the device itself. Therefore, the objective of the conventional optimal PMU placement problem was to find minimum number of devices, which when carefully placed throughout the network, maximised observability subject to different constraints. Due to improvements in relay technology, digital relays can now serve as both relays and PMUs. Under such circumstances, the substation installations consume the largest portion of the deployment cost, and not the devices themselves. Thus, for minimising cost of synchrophasor deployment, number of substation installations must be minimised. This study uses binary particle swarm optimisation to minimise number of substations in which installations must be performed for making all voltage levels observable, while being subject to various practical constraints. Standard IEEE systems have been used to explain the technique. Finally, a large-scale network of Dominion Virginia Power is used as the test bed for implementation.

Nomenclature

- $x(i)$: presence of phasor measurement unit (PMU) at $i$th bus
- $sub(k)$: presence of PMU at $k$th substation
- $sub\_bus(k)$: array of buses in $k$th substation
- $mb(i)$: measurement status of $i$th bus voltage
- $ml(j)$: measurement status of $j$th line current
- $fb(j)$: sending end bus number of $j$th line
- $rb(j)$: receiving end bus number of $j$th line
- $n_l$: number of lines
- $n_b$: number of buses in study area
- $n_{sub}$: number of substations
- $crit\_line$: critical lines
- $crit\_bus$: critical buses
- not allowed: set of all substations where installation is not possible
- $p$: particle/candidate solution for binary particle swarm optimisation (BPSO)
- $c_1, c_2$: cognition and social parameters for BPSO
- $itermax$: maximum number of iterations for BPSO
- $popsize$: population size for BPSO
- $r_1, r_2$: random numbers in $[0, 1]$

1 Introduction

For the power system to operate securely, reliable and accurate monitoring is necessary. This is done by making a variety of measurements throughout the network. The traditional measurements originated from the supervisory control and data acquisition (SCADA) system. Since the power system is spread over a large geographical area, time synchronised monitoring of the power system using only SCADA measurements was not possible. With the proliferation of phasor measurement unit (PMU) technology that provides highly accurate GPS synchronised phasor measurements, synchronised monitoring of the power grid’s bulk transmission network is now possible [1].

Many algorithms have been developed that use PMUs for state estimation, stability prediction, fault location, controlled islanding, situational awareness, and so on [2–6]. The primary requirement for most of these algorithms is that the system be completely observable by PMUs. However, because of the prohibitive cost of the device (largely due to the time synchronisation mechanisms), the main focus of previous research was on minimising the number of PMUs to be placed in the network while being subject to certain constraints. Some of the popular techniques that have been used for addressing the PMU placement problem were weighted least squares [7], integer programming [8–10], genetic algorithms [11, 12], Tabu search [13], and particle swarm optimisation [14]. Similarly, typical optimisation constraints have included measurement redundancy [15], criticality [16], inclusion of conventional measurements [17], and channel limitation [18]. An excellent summary of the state-of-the-art of the optimisation methods with respect to PMU placement can be found in [19].

All of the previous research seemed to minimise either the number of buses on which PMUs are to be installed or the number of PMU devices that were required for complete observability of the system. In a real system, there are multiple voltage levels (buses) at a particular substation and the tap settings between the different voltages are not usually known. Thus, different voltage levels are decoupled from the point of view of observability. It has also been proved that minimising the number of PMUs does not necessarily result in minimising the cost of PMU deployment [20]. Likewise, during an actual implementation at Dominion Virginia Power (DVP), a US-based utility, it was observed that the majority cost associated with synchrophasor deployment was not of the devices associated with synchrophasor deployment.
but rather of the outage of transmission equipment and labour costs. Considering this, the aim of the problem solved here is the minimisation of the total number of substation locations where installations are done so as to observe all the buses of the network. The effectiveness of binary particle swarm optimisation (BPSO) technique has previously been demonstrated in optimal allocation problems [21, 22] and it was employed here for solving this problem.

The rest of the paper is structured as follows. The problem formulation is explained in Section 2. A brief overview of BPSO is given in Section 3. The proposed solution methodology is described in Section 4. The results obtained on applying the proposed technique on a modified IEEE-14 bus system, a modified IEEE-30 bus system, and the DVP service area are presented in Section 5. A discussion of the practical relevance of the results is provided in Section 6. Finally, the conclusion is provided in Section 7.

2 Problem formulation

2.1 PMU installations in a practical system

A typical visualisation of a practical system with three voltage levels is shown in Fig. 1. The big black circles represent substations while the small black dots, triangles and rectangles represent buses at different voltage levels. Considering the most practical scenario, it is assumed that the transformer tap settings between different voltage levels are not known. Hence, the different shapes inside the black circles are not connected to each other. Now, in order to minimise number of substations if the big circles are assumed to be super-nodes, then according to the traditional optimal PMU placement (OPP) formulation, any circle in Fig. 1 that is connected to a circle with a PMU on it will be considered observed. On performing an integer programming solution based on such a logic the nodes selected for PMU placement come out to be 1, 2, and 10. However, from the figure it becomes clear that the 500 kV bus in substation 7 is not observed by this solution. Thus, from a practical implementation perspective, the OPP problem with the constraint of minimising number of substations cannot be solved by assuming super-node-based observability and so a different formulation is required.

The key points of the proposed formulation are as follows. (i) A bus at a particular voltage level that is monitored inside a substation should not translate to buses at other voltage levels becoming monitored inside the same substation. This is valid when transformer tap settings are not known beforehand, which is the case for most utilities. (ii) Different voltage levels are connected differently. This implies that the network given in Fig. 1 comprises of three separate networks (red, blue, and green) and not one. (iii) In a double bus-bar substation, if the buses at the same voltage level are connected by normally open switches, then they are to be treated as separate buses. (iv) The objective is to choose the minimum number of substations where PMUs would be installed so as to monitor different voltage levels. (v) Since considerable investment is made when placing a PMU at a substation [23], once a substation is chosen for PMU placement, all buses at that substation will have PMUs on them. In [24], the task of placing PMUs was done substation-wise. However, the intent in that paper was of minimising the number of devices, which does not represent the major portion of the cost [20, 23, 25].

2.2 Objective function

The problem herein is one of optimisation dealing with the minimisation of substations. The objective function is defined as shown in the following equation

\[ f = \sum_{k=1}^{n_{sub}} \text{sub}(k), \quad k \leq n_{sub} \]  (1)

In (1)

\[ \text{sub}(k) = \begin{cases} 1 & \text{if } x(i) = 1, \quad \forall i \in \text{sub}_{bus}(k) \\ 0 & \text{if } x(i) = 0, \quad \forall i \in \text{sub}_{bus}(k) \end{cases} \]  (2)

2.3 Constraints

2.3.1 Critical elements to be measured directly [16]: For the critical lines to be measured directly, a PMU needs to be installed on at least one of the ends of the line. Examples include tie-lines with neighbouring utilities. Thus, for a critical line \( j \)

\[ x(\text{fb}(j)) + x(\text{tb}(j)) \geq 1, \quad \forall j \in \text{crit}_{line} \]  (3)

Fig. 1 Visualisation of a practical system having multiple voltage levels; the dotted lines are at 765 kV, the dashed lines are at 345 kV, and the solid lines are at 500 kV.
For buses critical to the system such as high-voltage buses or buses with high connectivity, a PMU should be placed on each of them. Mathematically, this translates to
\[ x(i) = 1, \forall i \in \text{crit} \text{ bus} \]  
(4)

### 2.3.2 Full observability:
Before computing for full observability, some terms must be defined. A substation is a collection of buses. A bus is a collection of nodes that belong to the same voltage level and are present in the same substation (unless they are part of a double bus-bar substation and are connected by normally open switches, in which case they will be considered as separate buses). Most other PMU placement literatures use the terms bus and substation interchangeably and it is important to highlight the difference made here. A traditional PMU installed at a particular bus measures currents of all lines connected to that bus and voltage of that bus. A dual use line relay acting as PMU measures voltage of the bus at whose end it is deployed and line current for that particular line. Pseudo measurements are measurements which are derived from direct measurements obtained from PMUs. The rules governing pseudo measurements are:

(i) If the voltage at one end of the branch and the branch current are known, the voltage at the other end can be calculated using the branch parameters.
(ii) If the voltages at both ends of a branch are known, the branch current is also known.
(iii) Zero injection (ZI) buses are the buses with no injections. If current of only one branch that connects to a ZI bus is not known, then its value can be calculated from the other (known) branch currents. If only one of the buses directly connected to a ZI bus has an unknown voltage, it can be calculated if the voltage of the ZI bus is known.
(iv) In transformers which are under local control, the tap position is often not communicated to the control centre. This leads to additional errors in state estimation [26, 27]. In [28], an OPP problem with an added aim of estimating all transformer tap settings was introduced. In the proposed approach, all transformer branches are treated as branches having unknown impedances due to the tap settings being unknown.

Taking all this into account, the complete observability of the network can be represented mathematically as shown in the following equations
\[ \text{mb}(i) = 1, \forall i \leq n_b \]  
(5)
\[ \text{ml}(j) = 1, \forall j \leq n_l \]  
(6)

### 2.3.3 Prohibited substation installations:
PMUs need to communicate with the control centre and thus a high-speed communication medium is required. There are substations where PMU installations are not possible due to the inability or impracticality of installing fibre-optic or other types of high-speed communication. This can be incorporated into the optimisation by
\[ x(i) = 0, \forall i \in \text{sub} \text{ bus}(k), \forall k \in \text{not} \text{ allowed}(k) \]  
(7)

### 2.3.4 Grouping of buses in a substation:
Owing to the assumption of complete measurement of a substation when selected for PMU installation, if a bus in a substation is desired for PMU placement, all of the buses in that substation will obtain a PMU device. Mathematically, this can be stated as
\[ x(\text{sub} \text{ bus}(k)) = 1, \text{ if sub}(k) = 1 \]  
(8)

### 3 Binary particle swarm optimisation

Particle swarm optimisation [29] simulates the social behaviour of flocks of birds as they search for suitable habitat, food, and so on. A brief overview of it is presented as follows:

(i) Each candidate solution is called a particle, \( p \) and is represented by a \( d \) dimensional vector which gives its position in the \( d \) dimensional space. For example, in this case \( d \) is the number of buses in the system.
(ii) A set of such particles forms a swarm/population. The number of particles in a population is given by the variable popsize.
(iii) Each particle is associated with fitness, \( f \) which is a measure of particle quality. Generally, it will be the value of the function being maximised or minimised when evaluated for that particle.
(iv) The best position till the \( k \)th iteration of the \( i \)th particle is called ‘particle best’ and is denoted by \( p_{best}^{(i)} \). For a minimisation problem, this is updated at the end of each iteration as follows
\[ p_{best}^{(i)} = \begin{cases} p_i^{(k)}, & \text{if } f(p_i^{(k)}) < f(p_{best}^{(k-1)}) \\ p_{best}^{(k-1)}, & \text{otherwise} \end{cases} \]  
(9)

(v) The best position achieved till the \( k \)th iteration amongst all the particles is called ‘global best’ and is denoted by \( g_{best}^{(k)} \). This is given by
\[ g_{best}^{(k)} = \begin{cases} p_i^{(k)}, & \text{if } f(p_i^{(k)}) < f(g_{best}^{(k-1)}) \\ g_{best}^{(k-1)}, & \text{otherwise} \end{cases} \]  
(10)

(vi) There is also a velocity associated with each particle that gives its direction and step length of movement for successive iterations. At the \( k \)th iteration, the velocity in \( d \)th dimension of the \( i \)th particle is updated as shown in the following equation
\[ v_{id}^{(k)} = w \times v_{id}^{(k-1)} + c_1 \times r_1 \times (p_{best}^{(k-1)} - p_{id}^{(k-1)}) + c_2 \times r_2 \times (g_{best}^{(k-1)} - p_{id}^{(k-1)}) \]  
(11)

(vii) Finally, the position of the particle is updated as shown in the following equation
\[ p_{id}^{(k)} = p_{id}^{(k-1)} + v_{id}^{(k)} \]  
(12)

The binary version of particle swarm optimisation was introduced in [30]. The sigmoid function is used to contain the velocity in the \( d \)th dimension given by
\[ \text{sigmoid}(v_{id}^{(k)}) = \frac{1}{1 + e^{-v_{id}^{(k)}}} \]  
(13)

This is then used to discretise the position into binary. Thus, the position in the \( d \)th dimension is updated as shown in the following equation
\[ p_{id}^{(k)} = \begin{cases} 1, & \text{if } \text{rand} < \text{sigmoid}(v_{id}^{(k)}) \\ 0, & \text{otherwise} \end{cases} \]  
(14)
4 Solution methodology

4.1 Format of a particle

From Fig. 1 in Section 2.1 it can be realised that the number of substations cannot be minimised by reducing all the buses belonging to different voltage levels in the same substation into one entity and then treating it as a conventional OPP problem of minimising the number of entities where PMUs will be placed. This is because at each substation all types of voltage levels under study may not be present. Thus, the objective of the proposed methodology is to observe all the buses while minimising the number of substation installations. In accordance with this logic, a particle for solving the problem has the form shown in Fig. 2. The availability of PMU at a bus is represented by the value of the corresponding bit, which can be either 0 or 1. For example, if \( x_1 = 1 \), bus 1 has an installation that corresponds to the first bit in particle \( p \). The number of dimensions/bits is equal to the total number of buses present in the system.

4.2 Fitness of a particle

The fitness of a particle is given by (1) and is the objective function that needs to be minimised. The lower the value of the objective function for a particle, the better is its fitness.

4.3 Handling of constraints

When a particle changes position during iteration, it may leave the feasible region indicating that one or more constraints are not satisfied. Instead of regenerating the particle which would lead to a loss of computation done till the current iteration, a greedy-inspired repair algorithm is proposed that aims at making changes to the particle in order to satisfy the constraints as well as to accelerate the search for a global optimal point.

4.4 Greedy-inspired repair algorithm

The repair algorithm deals with constraints one by one and makes changes in each particle. A detailed description of the algorithm when operating on a particle is given below.

4.4.1 Prohibited substation installations: Step 1: Remove PMUs from prohibited substations in accordance with (7). These are the substations that lack communication infrastructure during the planning period because of which PMUs cannot be installed there.

4.4.2 Grouping of buses in a substation: Step 2: PMUs are installed on every bus of a substation with even one installation as given by (8).

4.4.3 Critical measurements: These comprise of buses and lines that must be measured directly and correspond to practical constraints described by (3) and (4). High-voltage buses, high-connectivity substations, tie-lines, and so on are examples of critical measurements. Steps 3–7 address this constraint.

Step 3: List all the critical measurements which have not been taken care of. If the array is empty, then GO TO Step 8.

Step 4: Form an appended array of allowed installation options for each critical measurement in terms of bus numbers at which PMU can be installed to satisfy it. Allow redundancy in the array, if any.

Step 5: Form an array of the corresponding substation number for each element of the array of installation options.

Step 6: Find the mode of this array to get the most common substation number and install PMUs on all buses of that substation.

4.5 Overall solution methodology

The final structure of the solution methodology is shown in the form of a flowchart in Fig. 3. In the simulations, multiple runs were done.

Step 7: Remove all the installation options (in terms of substation numbers) corresponding to the critical measurement(s) satisfied in current iteration from the array. If the array is not empty, then GO TO Step 6.

4.4.4 Full observability: Steps 8–12 ensure that the buses of all voltage levels are observed and correspond to practical constraints described by (5) and (6).

Step 8: For the current particle, list the unobserved buses using rules given in Section 2. If the array is empty, then GO TO Step 13.

Step 9: Form an appended array of allowed installation options for each unobserved bus in terms of bus numbers at which PMU can be installed to observe it. Allow redundancy in the array, if any.

Step 10: Form an array of the corresponding substation number for each element of the array of installation options.

Step 11: Find the mode of this array to get the most common substation number and install PMUs on all buses of that substation.

Step 12: Remove all the installation options (in terms of substation numbers) corresponding to the buses observed in the current iteration from the array. If the array is not empty, then GO TO Step 11.

Step 13: Stop.

Fig. 2 Particle for BPSO

Fig. 3 Flowchart depicting solution methodology
of the proposed technique with different initial particles (candidate solutions) to obtain the best result.

5 Results

The parameters for BPSO were taken as follows: $c_1 = c_2 = 2$, population size = 10, maximum iterations = 500.

5.1 Modified IEEE-14 bus system

The standard IEEE-14 bus system is modified into the type of system that is suitable for this problem. The modified system is shown in Fig. 4. Buses 1–5 belong to one voltage level, while buses 6–14 belong to a different voltage level. The transformer between buses 4 and 7 is removed. Substation A, which consists of buses 5 and 6 and substation B, which consists of buses 4 and 9 are multiple voltage level substations. The rest of the buses are single voltage level switchyards. Bus 7 is the ZI bus.

Results for optimal number of substations where PMUs are installed with no critical measurements or prohibited locations are given in Table 1. The results for the ‘conventional method’ are obtained using the logic proposed in [28]. It can be easily seen from the results obtained in Table 1 that minimising the number of buses with PMUs installed and minimising the number of substation locations with PMU installations are two separate objectives. Minimising the number of substation locations can lead to more number of PMUs being required (in this example, 4 instead of 3). However, this is desirable since the true goal is to reduce the cost by minimising the number of substation installations (2 instead of 3).

The results obtained when installations at substation B are not possible are shown in Table 2. It can be noted from Table 2 that as the grouping of buses on which PMUs are allowed to be placed starts decreasing, the optimal number of substation locations for PMU placement approaches the solution for traditional optimal PMU allocation.

5.2 Modified IEEE-30 bus system

Similar to the IEEE-14 bus system, the original IEEE-30 bus system is also modified as shown in Fig. 5. The model now comprises of three voltage levels as summarised in Table 3. Four two-winding transformers couple buses 4–12, 6–9, 6–10, and 27–28. Substations 21 and 24 are locations where PMU installations are assumed to be not practical. It is anticipated that in the near future dual use line relays acting as PMUs will be placed on lines connecting buses 9–11 on bus 9 side, 29–27 on bus 29 side, 30–27 on bus 30 side, 26–25 on bus 26 side, 15–14 on bus 15 side, 8–28 on bus 8 side, and 2–5 on bus 2 side. It is also assumed that due to a previous deployment process, a PMU is already located at substation 1. Lines connecting buses 12–14 and 15–23 are considered critical lines. Buses 5, 6, 9, 11, 12, 25, and 28 are ZI buses.

The results obtained are presented in Tables 4 and 5. As the locations and number of dual use line relays may change (since they are assumed to not have already been placed), two tables are provided – one in which they are present, and the other in which they are absent. The results indicate that if the dual use line relays are placed as planned, they will reduce the number of devices/ installations that are required for complete observability. From the tables it also becomes clear that although the proposed approach will result in a greater number of buses having PMUs on them, it will guarantee selection of a lesser number of substations where the installations must be done, which is a more desirable objective.

5.3 DVP transmission system

Now that the proposed method has been demonstrated on standard systems, it is applied to a real system. The system used is DVP’s network with voltage levels of 500, 230, 138, and 115 kV. The buses that belonged to the same substation and at the same voltage level were reduced into equivalent buses in accordance with Section 2.1. The resulting system had a total of 753 buses and 650 substations. There were a total of 866 non-transformer branches. All 28 tie-lines were taken as critical lines. The tap settings of all 79 transformers were assumed to be unknown. The reduced buses connected to a transformer with the other end not being of the voltage level under study were taken as buses with injections along with the ones with generations/loads. One hundred and eighty ZI buses were identified in the system. It was known beforehand that the 33 500 kV buses were fully observed. It was also speculated that 90 dual use line relay PMUs would be deployed throughout the transmission network over the next few years through standard substation upgrading procedures. This was included in the script for finding the list of unobserved buses. Also there were 77 substations where PMU installations were not possible over the planning horizon (3 years). This information was also included as a constraint in the optimisation. The results obtained for this system are given in Tables 6 and 7. Since ZI buses typically do not exist in practice, results for both the cases are provided (when they are considered and when they are not considered). From the tables, it can be realised that minimising the number of buses would result in a larger number of substations being covered, which would then incur heavier costs.

6 Discussion

This section analyses the practical implications of the results. Although a considerable drop was observed in the number of installations in Table 7 (line relay PMUs present) in comparison to Table 6 (line relay PMUs absent), it should be noted that the
results are specific to the DVP system. The reason for this is that the reduction in number depends entirely on the pattern that is followed for deploying those devices in the network. It may also happen in some systems that these relays do not significantly influence the optimal placement number. This will happen if they are added in parts of the network that are already observed due to previous deployment, in which case the added observability offered by them would only increase redundancy.

It is important to note that the objective of the proposed approach is complete observability of all bus voltages while minimising the number of substations where PMUs must be placed with the additional constraint that the tap settings are unknown. When the proposed algorithm selects a substation for PMU placement, all the buses inside that substation end up having PMUs on them, and so, the tap settings of transformer(s) located inside that substation can be calculated. However, the proposed algorithm is not designed to observe tap settings of all the transformers present in the system. This is because the substation’s internal consumption is its minimum demand and hence it cannot be neglected. It is for this reason that for the practical system (Section 5.3), results are shown

Table 3 Multiple voltage levels of modified IEEE-30 bus system

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Buses</th>
<th># Buses</th>
<th>Substations</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>1-8, 28</td>
<td>9</td>
<td>1-7, 22, 26</td>
</tr>
<tr>
<td>V2</td>
<td>10, 12-27, 29, 30</td>
<td>19</td>
<td>6, 7, 9-25</td>
</tr>
<tr>
<td>V3</td>
<td>9, 11</td>
<td>2</td>
<td>7, 8</td>
</tr>
</tbody>
</table>

Table 4 Results without dual use line relays acting as PMUs

<table>
<thead>
<tr>
<th>Objective</th>
<th>Solution</th>
<th>Number of buses</th>
<th>Number of substations</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimise devices (conventional)</td>
<td>buses: 6, 10, 11, 12, 18, 23, 27, 29 substations: 6, 7, 8, 16, 19, 22</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>minimise substations (proposed)</td>
<td>buses: 4, 6, 9, 10, 12, 18, 23, 27, 29 substations: 6, 7, 16, 19, 22</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5 Results with dual use line relays acting as PMUs

<table>
<thead>
<tr>
<th>Objective</th>
<th>Solution</th>
<th>Number of buses</th>
<th>Number of substations</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimise devices (conventional)</td>
<td>buses: 2, 10, 12, 15, 18 substations: 2, 7, 6, 15, 16</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>minimise substations (proposed)</td>
<td>buses: 4, 6, 9, 10, 12, 15, 20 substations: 6, 7, 12, 15</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>
both when ZI buses are considered as well as when they are not considered. Although BPSO is not guaranteed to always give optimal solution, it has the advantage of being flexible in terms of incorporating a wide variety of practical constraints and reaching solutions that are very close to the optimal solution. Moreover, since BPSO is a heuristic approach, the computational time required is much less in comparison to integer linear programming (ILP)-based approaches [31]. The greedy-inspired repair algorithm developed here was found to further increase the speed of the optimisation as is seen in Table 8. The computations were performed on an Intel (R) Core™ i7 Processor having a speed of 2.40 GHz and an installed memory (RAM) of 8 GB.

A Department of Energy (DOE) report published in October 2014 identified the communication infrastructure cost as the major portion of a PMU installation cost [23]. It said that in absence of adequate existing communications and/or upgrades to communication infrastructure, the cost of installing PMUs could increase by a factor of seven. However, it also stated that once a high-speed backbone telecommunications network was installed, the cost of installing additional PMUs dropped considerably. Labour and substation outage costs were also recognised as significant cost drivers. The report concluded by identifying that the PMU hardware cost was typically <5% of the total installed synchrophasor system cost. On comparing these observations with the proposed objective, the following inferences can be drawn: (i) by minimising substations where installations must be made, communication infrastructure costs are minimised directly; (ii) by minimising the number of locations to be covered, labour costs are minimised; and (iii) by minimising the number of substations (and by coordinating the PMU installations with planned outages), substation outage costs are minimised. Thus, these observations strongly justify application of the proposed technique for performing PMU installations in practical systems.

7 Conclusions

In a traditional PMU placement optimisation, the number of buses on which PMUs must be placed is minimised while being subject to various constraints. However, since the substation installation costs are much higher than the costs of the devices, a more practical solution is to minimise the number of substations that install PMUs. Moreover, as many substations have multiple voltage levels, the locations for optimal installations may be widely different for the two types of problems. In this paper, a method based on BPSO has been proposed for optimising the number of substations with installations to ensure complete network observability while satisfying other constraints such as critical measurements, prohibitive installations, and upgrading line relays to digital devices (dual use line relay PMUs).

Results obtained for standard IEEE systems demonstrated the ability of the proposed approach in handling the various constraints. It was observed that for the standard systems the global optima was reached at the end of the first iteration for every run. This was due to the implementation of a greedy-inspired repair algorithm that resulted in an inherent optimisation of the objective function thereby making the overall BPSO much faster. The method was finally implemented on a large-scale system of DVP. The analysis of the results indicates that the handling of practical constraints based on the concept of ‘substation minimisation’ is a revolutionary idea in the world of OPP designs.

8 Acknowledgments

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9 References


Table 6 Results for DVP system without dual use line relays acting as PMUs

<table>
<thead>
<tr>
<th>Objective</th>
<th>Not considering ZIs</th>
<th>Considering ZIs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of buses</td>
<td>Number of substations</td>
<td>Number of buses</td>
</tr>
<tr>
<td>minimise devices</td>
<td>minimise substations</td>
<td>minimise devices</td>
</tr>
<tr>
<td>277</td>
<td>251</td>
<td>244</td>
</tr>
<tr>
<td>299</td>
<td>233</td>
<td>266</td>
</tr>
</tbody>
</table>

Table 7 Results for DVP system with dual use line relays acting as PMUs

<table>
<thead>
<tr>
<th>Objective</th>
<th>Not considering ZIs</th>
<th>Considering ZIs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of buses</td>
<td>Number of substations</td>
<td>Number of buses</td>
</tr>
<tr>
<td>minimise devices</td>
<td>minimise substations</td>
<td>minimise devices</td>
</tr>
<tr>
<td>251</td>
<td>229</td>
<td>213</td>
</tr>
<tr>
<td>282</td>
<td>217</td>
<td>241</td>
</tr>
</tbody>
</table>

Table 8 Computation times for the standard test systems under different conditions

<table>
<thead>
<tr>
<th>Test system</th>
<th>Computation time, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using random repair algorithm</td>
<td>Using greedy-inspired repair algorithm</td>
</tr>
<tr>
<td>IEEE-14 bus system (all installations possible)</td>
<td>0.2028</td>
</tr>
<tr>
<td>IEEE-30 bus system (without dual use line relays)</td>
<td>3.9523</td>
</tr>
<tr>
<td>IEEE-30 bus system (with dual use line relays)</td>
<td>2.08</td>
</tr>
<tr>
<td>DVP system (with dual use line relays and ZI buses modelled)</td>
<td>5280</td>
</tr>
</tbody>
</table>