Unified PMU Placement Algorithm for Power Systems

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Abstract—This paper presents a PMU (phasor measurement unit) placement algorithm that gives the optimal PMU placement location and minimum number of PMUs for any electric network while considering different approaches. Complete observability, complete observability with N − 1 redundancy, different depths of unobservability, multi-stage sequential placement, real-time monitoring of critical buses are the various PMU placement approaches considered in this paper. The required input for the proposed algorithm is system information and other key aspects like locations of existing PMUs, locations unsuitable for PMU placement, and knowledge of critical buses. Binary integer programming is used to find the respective PMU placement location and minimum number of PMUs. The algorithm is tested on IEEE 39-bus system, IEEE 118-bus system, and IEEE 300-bus system. The results suggest that the algorithm can be used on any power system network and the suitable PMU placement approach can be implemented for application.

Keywords—Binary integer programming, observability, optimal placement, phasor measurement units (PMUs)

I. INTRODUCTION

A phasor measurement unit (PMU) or synchrophasor is a device which measures the electrical waves on an electricity grid, using a common time source for synchronization. The PMU measures the system state: voltage magnitude and angle of a particular location at a rate of multiple samples per second (typical value is 30 or 60 samples per second). Utilization of PMUs in the monitoring, protection and control of power systems has become increasingly important in recent years [1], [2].

All the benefits of PMUs can only be utilized if the PMUs are placed at correct locations. Observability of the system is the main consideration for PMU placement. Once the system is completely observable, complete picture of the system can be known using PMUs. As such, there has been lot of research on PMU placement [3], [4].

Optimal PMU placement (OPP) problem has been extensively researched since PMUs were first implemented. This problem can be addressed by two methods:

• Developing placement locations based on observability;
• PMUs are placed to represent critical system dynamics

References [5]–[10] have used the first approach whereas [11]–[16] have followed the second method. References [5], [6] solved the problem of OPP using integer linear programming (ILP) techniques based on binary search algorithms. Reference [7] introduced a two indices approach involving the system observability redundancy index and the bus observability index. References [8] and [9] achieved the objectives of minimizing the required number of PMUs and maximizing the measurement redundancy using integer quadratic programming and genetic algorithms, respectively. A unified approach of preserving system observability, and lowest system metering economy was developed in [10]. The second approach is used for voltage stability analysis [11]; voltage security assessment [12], [13]; transient stability prediction [14], and adaptive protection schemes [15], [16].

All the objectives described above are typically individually applied to the test systems. Because of that there is no PMU placement algorithm which integrates all the main placement techniques into one. The main objective of this paper is to find possible number of PMUs considering all scenarios with just the system data.

The rest of the paper is structured as follows. Section II explains the various PMU placement techniques considered in this algorithm. The proposed PMU placement algorithm is developed in Section III. Section IV demonstrates the application of this algorithm for IEEE 39-bus system, IEEE 118-bus system and IEEE 300-bus system. The conclusion is provided in Section V.

II. PMU PLACEMENT TECHNIQUES

IEEE 14-bus system is used to understand all PMU placement techniques. The first step is to find the bus-bus incidence matrix (A). It has the order N × N where N is the total number of buses. For IEEE 14-bus system, the A matrix is as shown in Fig. 1.

The rule for finding A matrix is as follows:

\[ A(i,j) = 1 \] if bus i is connected to bus j
\[ A(i,j) = 0 \] otherwise

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The rule for finding A matrix is as follows:

\[ A(i,j) = 1 \] if bus i is connected to bus j
\[ A(i,j) = 0 \] otherwise
Fig 1. Bus-Bus incidence matrix for IEEE 14-bus system

\[
A = 
\begin{bmatrix}
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
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0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

Fig 2. Signum \((A^2)\) matrix for IEEE 14-bus system

Now, \((A^{n+1})\) is depicts buses which are at most distance of \(n\) buses from each other. Fig. 2 shows the sign \((A^2)\) matrix which is used to find buses which are at most one bus away from each other for the IEEE 14-bus system. Using this information, the following Lemma can be stated.

Lemma 1: The \(ij\) entry in the \(n^{th}\) power of the incidence matrix for any graph or digraph is exactly the number of different paths of length \(n\), beginning at vertex \(i\) and ending at vertex \(j\) [4].

This is explained in Fig. 3 and the proof of Lemma 1 is given by induction [4]. The Signum of \(A^2\) is the incidence matrix of another graph which has branches added to the graph in Fig. 3. The dashed branches in Fig. 3 denote connections between nodes that pass through another node, i.e., those nodes are one step removed.

A. Complete observability

The main objective of PMU placement is that the system is completely observable using PMUs. Complete observability simply means that every bus in the system is monitored either directly or indirectly (i.e. by a PMU on an adjacent bus). The cost of PMU and associated cost of implementation make it necessary to have minimum number of PMUs for complete observability. This problem can be expressed mathematically as:

\[
\begin{align*}
\text{min} & \ f^T x \\
\text{subject to} & \ A x > 0 \quad x(i) = 0 \text{ or } 1 \\
f^T & = [1 1 1 1 \ldots 1] 
\end{align*}
\]

where \(A\) is the bus incidence matrix and \(x\) is the solution.

Using the above equation, by placing PMUs on buses 2, 6, 7, and 9, we can observe the system completely. This way we solve the OPP.

B. Complete observability with \(N-1\) redundancy

Complete observability provides us complete picture of the system with minimum number of PMUs, but if even one PMU is lost, we can lose many measurements. In power systems, it is very much possible that a PMU measurement can be lost due to various reasons. Hence it is usually recommended to have complete observability with \(N-1\) redundancy. It simply means that the system will be completely observable even if one PMU in the system is lost. Mathematically it says every bus should be monitored by at least 2 PMUs directly or indirectly as shown below:

\[
\begin{align*}
\text{min} & \ f^T x \\
\text{subject to} & \ A x > 1 \quad x(i) = 0 \text{ or } 1 \\
f^T & = [1 1 1 1 \ldots 1] 
\end{align*}
\]

This is a more reliable and robust PMU placement technique. For IEEE 14-bus system PMUs have to be placed on buses 2, 4, 5, 6, 7, 8, 9, 11 and 13. We can see that for 14 buses, we require 9 PMUs. Hence the disadvantage of this technique is that many PMUs are required for ensuring \(N-1\) redundancy for all the buses.

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C. Depth of unobservability

Incomplete observability means the voltage measurements of all the buses in the system are not available. That is, some buses are not monitored. It is done in order to find a trade-off for less number of PMUs. This can be done using the concept of depth of unobservability as explained below.

Concept of depth of unobservability: Depth D means a bus is at most D buses away from a directly or indirectly observed bus. Higher the value of D, lesser the number of PMUs required [3].

![Fig 4. Depth of one unobservability illustrated [3]](image)

Fig. 4 illustrates the one depth of unobservability. Bus D is one bus away from an indirectly observed bus E.

![Fig 5. Depth of two unobservability illustrated [3]](image)

Fig. 5 illustrates the two depth of unobservability. Bus D is two buses away from an indirectly observed bus F. Depth of unobservability is determined and solved using the following set of equations. For one depth of unobservability:

\[
\begin{align*}
\min f^T x_n \\
\text{subject to } & (A^D)x_n > 0 \\
& x_n(i) = 0 \text{ or } 1 \\
& f^T = [1 1 1 1 \ldots 1] \\
\text{constraint } & (f - x_1)^T x_2 = 0 \quad (D = 1)
\end{align*}
\]

For \(n^{th}\) depth of unobservability:

\[
\begin{align*}
\min f^T x_n \\
\text{subject to } & (A^{n+1})x_n > 0 \\
& x_n(i) = 0 \text{ or } 1 \\
& f^T = [1 1 1 1 \ldots 1] \\
\text{constraint } & (f - x_{n-1})^T x_n = 0 \quad (D = n)
\end{align*}
\]

D. Real time monitoring of critical buses

There are many buses in the system whose continuous monitoring is compulsory. Such buses are termed as critical buses of the system [17]. The following buses are identified as critical buses:

- High voltage buses
- High connectivity buses
- Buses relevant to transient/dynamic stability
- Control buses

The system will not be completely observable if depth of unobservability PMU placement technique is used. Moreover, we cannot let the critical buses be kept unobserved. Hence the first step is to install PMUs on all critical buses while simultaneously ensuring \(N - 1\) redundancy of critical buses and then implement depth of unobservability to find the other relevant PMU locations. Main advantage of this technique is that it takes care of observability of critical buses without significantly increasing the total number of PMUs required.

E. Multi-stage sequential technique

An electrical power system network consists of large number of buses. PMUs are needed on approximately 1/3rd number of total buses for complete observability [3], [15] and approximately 2/3rd number of total buses for complete observability with \(N - 1\) redundancy. Hence a large number of PMUs are required to be installed. These many PMUs cannot be installed at once. There are many economic as well as operational constraints that prevent that from happening. Hence multi-stage sequential technique needs to be implemented for PMU placement as shown below.

Suppose the targeted number of PMUs is to be placed in \(M\) stages, then the PMU placement will start from \(M - 1\) depth of unobservability. Then, for successive stages more number of PMUs will be installed according to reducing depths of unobservability. This technique gives the utility time and flexibility to place PMUs. The critical buses of the system can also be considered in this technique. PMUs on the critical buses can be placed first and then the placement can be started with \(M - 1\) depth of unobservability. The following set of equations is used to find number of PMUs for multi-stage sequential placement.

\[
\begin{align*}
\min f^T x_n \\
\text{subject to } & (A^{n+1})x_n > 0 \\
& x_n(i) = 0 \text{ or } 1 \\
& f^T = [1 1 1 1 \ldots 1] \\
\text{constraint } & (f - x_{n-1})^T x_n = 0 \quad (D = n)
\end{align*}
\]

In the above equation, \(n\) starts from \(M - 1\) (where \(M\) is the number of stages) and goes to 1. The solution is \(x_n\).

III. PROPOSED ALGORITHM

A. Network Data

1. Basic Network Data

The basic network data required is the ‘From’ bus and ‘To’ bus information. Using this data, A matrix is calculated and the equations are used to solve for number of PMUs and their locations.
b) Existing PMUs
The next step is to check if there are any existing PMUs on the system. If any existing PMUs are present in the system then along with the equations for respective technique we add the following constraint

\[ (f - x_e)^T x_n = 0 \]

where \( x_e \) is a matrix showing existing PMUs.

c) Critical buses
The critical buses in the system are identified according to the rule stated above. It is integrated into the system along with the respective equations using the following constraint

\[ (f - x_c)^T x_n = 0 \]

where \( x_c \) is a matrix showing PMUs on critical buses.

d) No PMUs on some buses
In some cases there are conditions where no PMUs can be placed on particular buses due to various reasons. It is integrated in system along with respective equations using the following constraint

\[ (f - x_n0)^T x_n = 0 \]

where \( x_n0 \) is a matrix showing buses where PMUs cannot be placed.

B. Binary Integer Programming
The set of equations for OPP using all techniques are solved using binary integer programming. Binary integer programming solves the problem

\[
\min f^T x \quad \text{subject to: } A x \leq b, \\
Aeq x = beq \\
\text{where } x(i) = 0 \text{ or } 1 \\
f^T = [1 \ 1 \ 1 \ 1 \ \ldots \ \ldots \ 1]
\]

The solution of binary integer programming is the matrix \( x \) which gives the number of PMUs and their locations. The total number of PMUs is given by \( f^T x \). The respective PMU locations can be found by checking the rows of \( x \) which are 1. The row number corresponds to the Bus number of the system. The observability of the system can be evaluated using \( A \times x \). The element corresponding to each row of \( A \times x \) is the number of observability for that respective bus.

Fig. 6 shows the flowchart for unified PMU placement algorithm considering various scenarios discussed above. The first step is getting all the system data as described above. Next step is to select the PMU placement technique according to requirement. Finally the set of equations for respective technique along with constraints if any is solved using binary integer programming.

IV. EXAMPLE TEST SYSTEMS
A. IEEE 39-bus system
Assumptions:
Existing PMUs: 6, 16
Critical buses: 10, 23, 39
No PMUs on some bus: 21, 28

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Table I shows the results when the proposed algorithm is applied to IEEE 39-bus system. The existing PMU locations are assumed to be the high voltage buses and the critical buses are high connectivity buses. No PMU buses are randomly selected low-voltage buses where PMUs cannot be placed due to some constraints.

**B. IEEE 118-bus system**

Assumptions:
Existing PMUs: 8, 9, 10, 26, 30, 38, 63, 64, 65, 68, 81
Critical buses: 11, 12, 49, 66, 80, 92, 100
No PMUs on some bus: 3, 5, 49

Table II shows the results when the proposed algorithm is applied to IEEE 118-bus system. The existing PMU locations are assumed to be the high voltage buses and the critical buses are high connectivity buses. No PMU buses are randomly selected low-voltage buses where PMUs cannot be placed due to some constraints.

<table>
<thead>
<tr>
<th>CASES</th>
<th>#PMUs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>COMPLETE OBSERVABILITY</strong></td>
<td></td>
</tr>
<tr>
<td>No Existing PMUs</td>
<td>13</td>
</tr>
<tr>
<td>With Existing PMUs on some buses</td>
<td>14</td>
</tr>
<tr>
<td>With No PMUs on some buses</td>
<td>14</td>
</tr>
<tr>
<td>N-1 Redundancy with complete observability</td>
<td></td>
</tr>
<tr>
<td>No Existing PMUs</td>
<td>28</td>
</tr>
<tr>
<td>With Existing PMUs on some buses</td>
<td>28</td>
</tr>
<tr>
<td>Depth of Unobservability (DoU)</td>
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</tr>
<tr>
<td>DoU=1</td>
<td>7</td>
</tr>
<tr>
<td>DoU=2</td>
<td>5</td>
</tr>
<tr>
<td>DoU=3</td>
<td>3</td>
</tr>
<tr>
<td>With Existing PMUs on some buses</td>
<td></td>
</tr>
<tr>
<td>DoU=1</td>
<td>8</td>
</tr>
<tr>
<td>DoU=2</td>
<td>4</td>
</tr>
<tr>
<td>DoU=3</td>
<td>3</td>
</tr>
<tr>
<td>Depth of Unobservability (DoU) for critical buses</td>
<td></td>
</tr>
<tr>
<td>DoU=0</td>
<td>15</td>
</tr>
<tr>
<td>DoU=1</td>
<td>8</td>
</tr>
<tr>
<td>DoU=2</td>
<td>6</td>
</tr>
<tr>
<td>DoU=3</td>
<td>6</td>
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</table>

<table>
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<tr>
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<tr>
<td><strong>COMPLETE OBSERVABILITY</strong></td>
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<td>No Existing PMUs</td>
<td>68</td>
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<tr>
<td>With Existing PMUs on some buses</td>
<td>72</td>
</tr>
<tr>
<td>Depth of Unobservability (DoU)</td>
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<tr>
<td>DoU=1</td>
<td>16</td>
</tr>
<tr>
<td>DoU=2</td>
<td>8</td>
</tr>
<tr>
<td>DoU=3</td>
<td>7</td>
</tr>
<tr>
<td>With Existing PMUs on some buses</td>
<td></td>
</tr>
<tr>
<td>DoU=1</td>
<td>23</td>
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<tr>
<td>DoU=2</td>
<td>17</td>
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<tr>
<td>Depth of Unobservability (DoU) for critical buses</td>
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<td>DoU=0</td>
<td>41</td>
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<tr>
<td>DoU=1</td>
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</tr>
<tr>
<td>DoU=2</td>
<td>22</td>
</tr>
<tr>
<td>DoU=3</td>
<td>18</td>
</tr>
</tbody>
</table>

C. **IEEE 300-bus system**

Assumptions:
Existing PMUs: 4, 16, 28, 33, 36, 40, 68, 169, 173, 174, 198, 210, 216, 242
Critical buses: 31, 109, 190, 15
No PMUs on some bus: 3, 5, 49

Table III shows the results when the proposed algorithm is applied to IEEE 300-bus system. The existing PMU locations are assumed to be the high voltage buses and the critical buses are high connectivity buses. No PMU buses are randomly selected low-voltage buses where PMUs cannot be placed due to some constraints.

V. **CONCLUSION**

A unified PMU placement algorithm is developed in this paper considering various PMU placement techniques. Complete observability gives the minimum number of PMUs but is not robust. Complete observability with N – 1 redundancy gives a robust solution but the number of PMUs required is quite large. Depth of unobservability is used when less number of PMUs is to be installed in the system.
Multi-stage sequential technique gives time and flexibility to place PMUs on the system. The algorithm successfully gives the number of PMUs and their respective location for all the placement techniques discussed in this paper. The sample test cases prove it. The algorithm can be effectively used for PMU placement on any electrical network.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>NUMBER OF PMUS FOR IEEE 300-BUS SYSTEM CONSIDERING VARIOUS SCENARIOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASES</td>
<td>#PMUs</td>
</tr>
<tr>
<td>COMPLETE OBSERVABILITY</td>
<td></td>
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<tr>
<td>No Existing PMUs</td>
<td>65</td>
</tr>
<tr>
<td>With Existing PMUs on some buses</td>
<td>73</td>
</tr>
<tr>
<td>With No PMUs on some buses</td>
<td>73</td>
</tr>
<tr>
<td>N-I Redundancy with complete observability</td>
<td></td>
</tr>
<tr>
<td>No Existing PMUs</td>
<td>155</td>
</tr>
<tr>
<td>With Existing PMUs on some buses</td>
<td>159</td>
</tr>
<tr>
<td>Depth of Unobservability (DoU)</td>
<td></td>
</tr>
<tr>
<td>No Existing PMUs</td>
<td></td>
</tr>
<tr>
<td>DoU=1</td>
<td>31</td>
</tr>
<tr>
<td>DoU=2</td>
<td>16</td>
</tr>
<tr>
<td>DoU=3</td>
<td>9</td>
</tr>
<tr>
<td>With Existing PMUs on some buses</td>
<td></td>
</tr>
<tr>
<td>DoU=1</td>
<td>40</td>
</tr>
<tr>
<td>DoU=2</td>
<td>25</td>
</tr>
<tr>
<td>DoU=3</td>
<td>20</td>
</tr>
<tr>
<td>Depth of Unobservability (DoU) for critical buses</td>
<td></td>
</tr>
<tr>
<td>DoU=0</td>
<td>76</td>
</tr>
<tr>
<td>DoU=1</td>
<td>42</td>
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<tr>
<td>DoU=2</td>
<td>28</td>
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<tr>
<td>DoU=3</td>
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REFERENCES


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